# MTM3691-Theory of Linear Programming

Gökhan Göksu, PhD

Week 1

# Contact Information and Course Evaluation Criteria

| Gökhan Göksu, PhD | |
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- Midterm: 35 %
- Quiz: 25 %
- Final Exam: 40 %
- Attendance will not affect your grade!

## Course Content and Textbooks

- Chapter 2: Introduction to Linear Programming (LP)
  - Two-Variable LP Model
  - Characteristics of LP Model
  - Graphic Solution Method (Max/Min)
- Chapter 3: The Simplex Method
- Chapter 4: Duality and Sensitivity Analysis
- Chapter 5: Transportation Model and Various Transportation Models

📕 H. A. Taha, Operations Research: An Introduction, 8/e, Pearson Education, 2008.

📕 H. A. Taha, Çeviren ve Uyarlayanlar: Ş. A. Baray, Ş. Esnaf, Yöneylem Araştırması, 6. Basımdan Çeviri, Literatür Yayınevi, 2000.

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# What is "Linear Programming"?

- In mathematics, especially in the applied branch of operations research, linear programming problems are the optimization (i.e. minimization or maximization of the objective function) of a linear objective function in a way that satisfies the linear equality and/or inequality constraints.

- An optimization model is called a linear program if it has continuous variables, a single linear objective function and all its constraints consist of linear equations or inequalities.

- In other words, the single-objective function and all constraints of the model should consist of weighted sums of continuous decision variables.

# Terminology

- The word linear in linear programming indicates that all functions in the model must be linear.

- The word programming does not refer to "computer programming"; rather it is synonymous with "planning".

- Therefore, linear programming involves planning activities that will find the optimal solution that fits a specified goal among many suitable alternatives.

# History

- The study of a problem in the form of a system of linear inequalities dates back to the work of Joseph Fourier and is named the Fourier-Motzkin elimination method in memory of this famous mathematician.

- While all economic planning issues become prominent "in practice" in Soviet Russia in the 1920s, among the theoretical studies carried out to show how the entire economy could be planned theoretically, Leonid Kantorovich's contribution led to a linear programming problem being clearly revealed for the first time.

- A research group lead by George Dantzig established to examine the logistics allocation problems that arose in the United States during World War II, revealed the need to define the linear programming problem to solve such problems.

# History

- To solve linear programming problems, they introduced a solution methodology called the "simplex algorithm".

- Especially since it was clearly seen that this mathematical model and solution algorithm led to the reduction of war expenses by planning costs and returns, these theoretical and practical developments remained a state secret until 1947.

- In 1947, John von Neumann, while also particularly interested in the theory of games, developed the theory of duality.

- The Nobel Prize in Economics was awarded to Kantorovic, Dantzig and von Neumann in 1975 for their contributions to linear programming.

# The Areas of Use

- Military Applications,
- Networks,
- Supply Chains,
- Production Planning,
- Human Resources Planning,
- Logistics,
- Agriculture,

- Transportation
- Telecommunications
- Business, Economics,
- Inventory Control,
- Energy Planning,
- Marketing,
- …

# Formulation

- Mathematically, a linear programming problem has three fundamental components.

- **Objective function** - An objective function, to be minimized or maximized

  - For an n dimensional problem:

    $$\max \quad c_1 x_1 + c_2 x_2 + \cdots + c_n x_n \quad \text{or}$$

    $$\max \quad \mathbf{c}^T \mathbf{x}$$

  - For a 2 dimensional problem:

    $$\max \quad c_1 x_1 + c_2 x_2$$

# Formulation

- Mathematically, a linear programming problem has three fundamental components.

- **Constraints** - Constraints, in the form of equality or inequality

    - For a problem with m variables and n constraints:

    $$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$

    $$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

    $$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

    $$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

    or

    $$A\mathbf{x} \leq \mathbf{b}$$

# Formulation

- Mathematically, a linear programming problem has three fundamental components.

- **Constraints** - Constraints, in the form of equality or inequality
  - For a problem with 2 variables and 3 constraints:

$$a_{11} x_1 + a_{12} x_2 \leq b_1$$
$$a_{21} x_1 + a_{22} x_2 \leq b_2$$
$$a_{31} x_1 + a_{32} x_2 \leq b_3$$

# Formulation

- Mathematically, a linear programming problem has three fundamental components.

- **Nonnegativity constraints** – all the variables should be greater or equal to 0
  - For an n dimensional problem:

  $$x_1 \geq 0, x_2 \geq 0 ..... x_n \geq 0$$

  or

  $$\mathbf{x} \geq 0$$

  - For a 2 dimensional problem:

  $$x_1 \geq 0 \quad \text{and} \quad x_2 \geq 0$$

# Formulation

- All the components can be written in terms of vector and matrix notation as

$$\max \quad \mathbf{c}^T \mathbf{x}$$

$$\text{subject to} \quad \mathbf{Ax} \leq \mathbf{b}, \ \mathbf{x} \geq 0$$