MATLAB Tutorial Session #1
MTM4502-Optimization Techniques
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Main Task

Write a MATLAB program to find the minimum of the function

$$f(\mathbf{x}) = 0.25x_1^4 - 0.5x_1^2 + 0.1x_1 + 0.5x_2^2$$

subject to $-2 \leq x_i \leq 2$, for $i = 1, 2$, by using Newton-Raphson and Hestenes-Stiefel algorithms. Repeat the main steps of your algorithms until the desired accuracy is achieved, i.e.

$$\|\nabla f(\mathbf{x}_k)\| \leq \epsilon \text{ (or } |f(\mathbf{x}_{k+1}) - f(\mathbf{x}_k)| \leq \epsilon).$$

Take the initial guess as $\mathbf{x}_0 = \begin{bmatrix} -1 & 1 \end{bmatrix}^\top$ and absolute error bound as $\epsilon = 10^{-4}$ for every algorithm. The global minimum is located at $\mathbf{x}^* = \begin{bmatrix} -1.0465 & 0 \end{bmatrix}^\top$, $f(\mathbf{x}^*) \approx -0.3523$. You may see sample realizations of these algorithms as the following.

```
Newton-Raphson Algorithm
k=1, x1=-1.000000, x2=1.000000, f(x)=0.150000
k=2, x1=-1.050000, x2=0.000000, f(x)=-0.352373, abs. error=0.502373
...
Elapsed time is ... seconds.
Hestenes-Stiefel Algorithm
k=1, x1=-1.000000, x2=1.000000, f(x)=0.150000
k=2, x1=-1.099000, x2=0.010000, f(x)=-0.349055, abs. error=0.499055
...
Elapsed time is ... seconds.
```

Answer the following questions:

- How many steps does it take to find the minimum of this function with both of these algorithms?
- What are the execution times of these algorithms? Does this make sense?
- Does the convergence depend on the initial conditions? Why?
- Based on the last two questions, what can be the reason for this trade-off?
- Do you expect the same number of steps and execution times, when you change the stopping criterion and the absolute error bound?

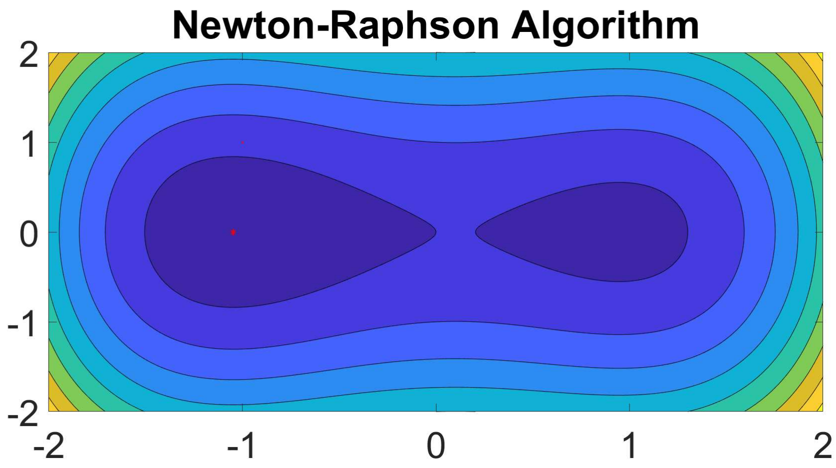

Figure 1: Newton-Raphson Algorithm.

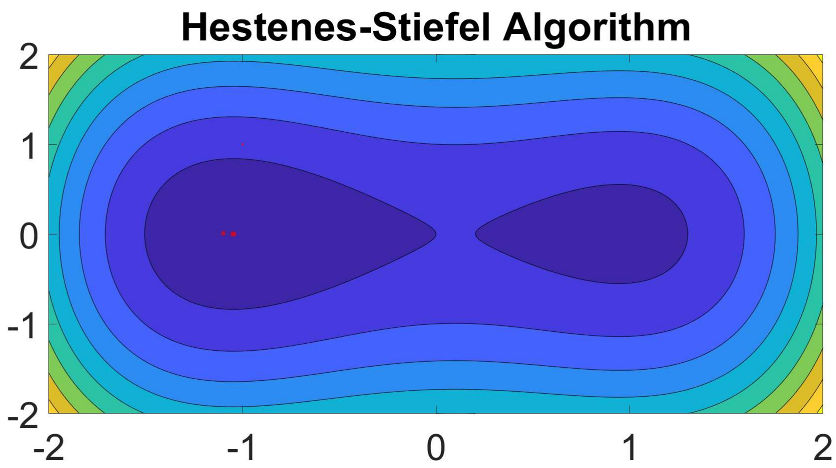

Figure 2: Hestenes-Stiefel Algorithm.