MTM4502-Optimization Techniques Term Project Date: 14/04/2025
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Parameter Selection

Take the parameters a and b as the last two digits of your student number. For instance, if your student number is 18058063, take $a = 6$ and $b = 3$.

Introduction

Objective functions plays an important role to validate and compare the performance of optimization algorithms. Many benchmark or test functions have been documented in various publications, but there isn't a universally accepted list of benchmark functions. The most effective test functions should possess a range of distinct characteristics, allowing for unbiased testing of new algorithms. In [1], a rich set of 50 benchmark functions for unconstrained optimization problems with diverse properties are presented. In the course, as you know, we made a tutorial session for

$$f_{02}(\mathbf{x}) = 0.25x_1^4 - 0.5x_1^2 + 0.1x_1 + 0.5x_2^2$$

subject to $-2 \leq x_i \leq 2$, for $i = 1, 2$, by using Newton-Raphson and Hestenes-Stiefel algorithms.

Main Task

Write a MATLAB program to find the minimum of the function $f_{cd}(\cdot)$ where $cd = ab$ (mod50) from [1, Appendix B] by using

- Newton-Raphson,
- Hestenes-Stiefel,
- Polak-Ribière and
- Fletcher-Reeves algorithms.

You are also strongly encouraged to use another relevant algorithm from the literature, which will be rewarded with an extra 10 points. If the function is not differentiable, use an approximation proposed by yourself or an relevant approximation commonly used in the literature and write it explicitly in your report. Repeat the main steps of your algorithms until the desired accuracy is achieved, i.e.

$$\|\nabla f(\mathbf{x}_k)\| \leq \epsilon \text{ and } |f(\mathbf{x}_{k+1}) - f(\mathbf{x}_k)| \leq \epsilon.$$

Take **<u>THREE</u>** initial guess as $\mathbf{x}_0 \sim \mathcal{N}_n(0, 1)$ (n-dimensional vector having elements from standard normal distribution by using `randn` function of MATLAB) or $\mathbf{x}_0 \sim \mathcal{U}_n(x_{0,\min}, x_{0,\max})$ (n-dimensional vector having elements from uniform distribution from the closed interval $[x_{0,\min}, x_{0,\max}]$ where $x_{0,\min}$ and $x_{0,\max}$ are specified for each function in [1] by using `rand` function of MATLAB). For instance, if your problem is defined on $-2 \leq x_i \leq 2$, for $i = 1, 2$, you may consider to choose $\mathbf{x}_0 \sim \mathcal{U}_n(-2, 2)$. Take also the absolute error bound as $\epsilon = 10^{-4}$ for every algorithm.

Instructions

Write a project report regarding to the given task above.

- [10p] Start introducing the corresponding function,
- [10p] Find the stationary points **NUMERICALLY** and classify them by using the second order sufficient conditions.

Write the project report regarding to the given task by answering the following questions (please **EXPLAIN** all of them by at least two sentences, out of 50 points):

- How many steps does it take to find the minimum of this function with all of these algorithms?
- What are the execution times of these algorithms? Does this make sense?
- Does the convergence depend on the initial conditions? Why?
- Based on the last two questions, what can be the reason for this trade-off?
- Do you expect the same number of steps and execution times, when you change the stopping criterion and the absolute error bound?

Your project report also **MUST**

- [10p] contain at least one figure,
  - if your problem is 2-dimensional, then your figure must include the all the steps starting from **THREE** random initial points with **DIFFERENT COLORS** and all the steps corresponding to the same iteration **MUST** be plotted with the **SAME** **COLOR**!),
  - if your problem has three or more dimensions, then you should include at least one **MEANINGFUL** figure. Some examples may include (but not limited to!) evolution/convergence of norms of points, performance analysis (i.e. sensitivity of number of iterations or execution times with respect to various initial conditions, error bounds or stopping criteria).
- [10p] contain at least one table for benchmark,
- [10p] be at least two pages long and be written by IEEE conference proceeding template [2]!
- **USE 4 SIGNIFICANT FIGURES AFTER THE DECIMAL POINT!** Once not done, it will be subject to a 10-point deduction!

Submission Information

All projects must be uploaded via `online.yildiz.edu.tr` with

- **ONE SINGLE PDF** document,
- ***Codes.zip*** file, including **EXECUTABLE** *.m files:*
  - *Nonlinear.m*,
  - *func.m*,
  - *gradfunc.m*,
  - *hessianfunc.m*

Please **DO NOT EMBED** your codes directly into your report (i.e. the PDF document! Once done, it will be subject to a 50-point deduction!!!). **Any project submitted after the deadline will be subject to a 25-point deduction per 12 hour period**. Collaboration is strongly recommended and you may work with your friends. You may also use AI tools, but at the end, it should be a **MEANINGFUL** project report and each student should be able to present his/her program, project and report. Don't expect points from the **MEANINGLESS** comments that AI tools generates.

References

[1] M. M. Ali, C. Khompatraporn, and Z. B. Zabinsky, "A numerical evaluation of several stochastic algorithms on selected continuous global optimization test problems," *Journal of Global Optimization*, vol. 31, no. 4, p. 635, 2005.

[2] IEEE, "Manuscript Templates for Conference Proceedings." `https://www.ieee.org/conferences/publishing/templates.html`. Accessed: 2023-05-04.